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RESEARCH ARTICLE

THE IMPACT OF MEMORY EFFECT AND NONLOCALLITY IN COVID-19 WORLD DATA USING HYBRID FRACTIONAL ORDER COMPARTMENTAL MODEL AND NEURAL NETWORKS

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Abstract

Background: This study investigates the impact of memory effects and nonlocality on COVID-19 World Data. The primary objective is to explore the dynamics of the pandemic using a hybrid fractional order compartmental model combined with neural networks. **Methods**: The research employs a hybrid fractional order compartmental model alongside an artificial neural network. Key procedures include stability analysis of equilibrium points, the development of a Disease Informed Neural Network (DINN) by integrating the fractional order model with neural networks, and the application of Laplace Transforms to expedite fractional derivative computations during neural network training.
Results: The study identifies optimal fractional order values: $\alpha_1 = 0.7899$, $\alpha_2 = 0.8636, \alpha_3 = 0.8496$, and $\alpha_4 = 0.8591$. The disease transmission parameters are determined as $\rho = 0.1730, \delta = 0.0466$, and $\omega = 0.0018$. Numerical simulations are conducted, which visually compare the hybrid fractional order compartmental model and neural network results against real COVID-19 World Data across all compartments. **Conclusion**: The paper concludes that the developed model effectively captures the dynamics of COVID-19, emphasizing the role of memory and nonlocal effects in disease transmission. The insights gained from plotting dynamic model behaviors, including minimum and maximum solutions, contribute to a comprehensive understanding of disease transmission and inform potential interventions.

Keywords: *Memory Effects, Fractional Order Compartmental Model, COVID-19, Disease Informed Neural Network*
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INTRODUCTION

Background

The outbreak of the COVID-19 pandemic highlighted the urgent need for accurate and reliable disease forecasting models. In light of the increasing threat of human annihilation as a result of pandemics, it is crucial to delve into the mathematical models of infectious diseases. This undertaking is indispensable for obtaining a more profound understanding of their transmission dynamics and controls. The mathematical study of COVID-19 pandemic cut across several models including agent-based (Cuevas 2020; Kano et al. 2021; Kasereka et al. 2023; Kerr et al. 2021; Krivorotko, Sosnovskaia, and Kabanikhin 2023; Silva et al. 2020), compartmental, network (Calvetti et al. 2020; Chung and Chew 2021; Huang et al. 2020; Milić, Milojković, and Jeremić 2022) and spatial models (Barnes et al. 2022; Booton et al. 2021; Clement et al. 2021; Danon et al. 2021; Davidson and Wainer 2021; Maza and Hierro 2022). In search of accuracy many complexities such as delays (Arora et al. 2023; Babasola et al. 2022; Scheiner, Ukaj, and Hellmich 2020), government interventions (Caldwell et al. 2021; Dwomoh et al. 2021; Fang, Nie, and Penny 2020; Mahikul et al.

2021), hospitalization (Asteris et al. 2022; Gu et al. 2022; Mohammed et al. 2022; Pasco et al. 2020; Rahmani et al. 2022; Srivastav et al. 2021a), optimal controls (Abioye et al. 2021; Araz 2021; Ayalew et al. 2023; Kouidere et al. 2021; Ojo et al. 2022; Salama et al. 2023; Sasmita et al. 2020; Shen et al. 2021; Tu et al. 2023; Zamir et al. 2021) population structure (Arisi and Mantuano 2020; Blyuss and Kyrychko 2021; Dai et al. 2023; Jimenez-Rodriguez et al. 2022; Ladner et al. 2020; Tilstra et al. 2023; Yu 2020), quarantine (Aba Oud et al. 2021; Babaei et al. 2021; Fredj and Chérif 2020; Gu et al. 2022; Memon, Qureshi, and Memon 2021; Pandey et al. 2021; Prathumwan, Trachoo, and Chaiya 2020; Srivastav et al. 2021a), social distance (Badr et al. 2020; Chandra, Singh, and Bajpai 2021; Chen et al. 2021; Choi and Ki 2020; Kim et al. 2020; Moyles, Heffernan, and Kong 2021; Mwalili et al. 2020), stochastic variations (Ali and Khan 2023; Chatterjee et al. 2020; Noor et al. 2022; Srivastav et al. 2021b; Thul and Powell 2023; Zhang et al. 2020) vaccination (Aldila et al. 2021; Aruffo et al. 2022; Ayalew et al. 2023; Bhatter et al. 2023; Diagne et al. 2021; Jentsch, Anand, and Bauch 2021; Kahn et al. 2022; Moore et al. 2021; Olivares and Staffetti 2021; Shen et al. 2021; Wagner, Saad-Roy, and Grenfell 2022; Watson et al. 2022; Yang, Yu, and Cai 2022; Yavuz et al. 2021) and waves (Anitha et al. n.d.; Awasthi 2023; González-Parra and Arenas 2023; Hussain et al. 2022; Kuwahara and Bauch 2023; Lobato, Libotte, and Platt 2021; Mandal et al. 2021; Martínez-Fernández et al. 2023; Omede et al. 2023; Singh et al. 2023; Zine et al. 2020) were captured. Fractional calculus in other fields has shown promising results in predicting complex systems . Due to the limitations of classical mathematical models in accurately representing these diseases, the implementation of fractional differential equations has emerged as a solution to address these issues (Podlubny 1999).

Review of Literature:

The advancement of technology and the availability of large-scale data, new approaches are emerging to enhance these models' accuracy and predictive power. One such approach is disease-informed neural networks (DINN) (Shaier, Raissi, and Seshaiyer 2021). These neural networks combine the strengths of traditional mathematical models and artificial intelligence techniques to improve the accuracy of disease forecasting.

With the availability of data, we will consider compartmental model of COVID-19 (Nisar et al. 2021) where total population N is divided into four epidemiological compartments: $\mathcal{U}$ (susceptible compartment), $\mathcal{V}$ (infected compartment), $\mathcal{P}$ (recovered compartment), and $\mathcal{Q}$ (death compartment).

$$\begin{aligned} \dot{\mathcal{U}}(t) &= -\rho\frac{\mathcal{U}(t)\mathcal{V}(t)}{N}, \\ \dot{\mathcal{V}}(t) &= \rho\frac{\mathcal{V}(t)\mathcal{U}(t)}{N} - (\delta+\omega)\mathcal{V}(t), \\ \dot{\mathcal{P}}(t) &= \delta\mathcal{V}(t), \\ \dot{\mathcal{Q}}(t) &= \omega\mathcal{V}(t). \end{aligned} \tag{1}$$

In the model, ρ represents the average number of contacts per person per time, while δ is the recovery rate and ω stands for the death rate.

In addition to DINN, fractional order compartmental models have also been applied in studying the COVID-19 epidemic (A.A. Alderremy et al. 2020; Ahmed Boudaoui et al. 2021; Amar Nath Chatterjee and Bashir Ahmad 2021; Ebrahem A. Algehyne and Muhammad Ibrahim 2021; Faïçal Ndaïrou et al. 2021; Kamal Shah et al. 2021; Min Cai, George Em Karniadakis, and Changpin Li 2022; Olumuyiwa J. Peter et al. 2021; Pushpendra Kumar and Vedat Suat Erturk 2020; Samuel Okyere, Ebenezer Bonyah, and Joseph Ackora Prah 2022; Shabir Ahmad et al. 2020; S.S. Askar et al. 2021; Zizhen Zhang et al. 2020). Fractional order models introduce fractional derivatives into the traditional framework, allowing for a more precise description of the transmission dynamics with nonlocality and memory effects. These models consider that individuals may exhibit different levels of susceptibility, exposure, and infectivity over time. Hybrid fractional order models and neural networks have proven to be valuable tools in understanding and predicting the pandemic and making informed policies regarding control measures and resource allocation. These approaches have been used in various studies to simulate and forecast the evolution of COVID-19, analyze the spread of the virus under different scenarios, and assess the effectiveness of different control measures (Cai, Em Karniadakis, and Li 2022; De Rosa et al. 2023; Grimm et al. 2022; Han et al. 2023; Ke, Ma, and Yin 2022; Kharazmi et al. 2021; Linka et al. 2022; Long, Khaliq, and Furati 2021; Malinzi, Gwebu, and Motsa 2022; Mattheakis et al. 2022; Nguyen, Raissi, and Seshaiyer 2022; Ning et al. 2022, 2023; Panicker et al. 2021; Rodríguez et al. 2023; Schiassi et al. 2021; Shaier et al. 2021; Torku, Khaliq, and Furati 2021; Treibert and Ehrhardt 2021, 2022; Wu et al. 2022; Zizhen Zhang et al. 2020).

Study Aim:

In this work, the fractional order Model (1) is hybridized with Neural Networks, and the resulting algorithm is applied to estimate fractional and disease parameters using COVID-19 world data. The dynamics of COVID-19 were learned using this hybrid algorithm, followed by predictions. Additionally, the solution spectrum was analyzed in relation to fractional parameters, enhancing our understanding of disease transmission and the effects of memory and nonlocality.

The article is organized in the subsequent manner: Section 2 explores the basic concepts, whereas section 3 introduces the hybridized fractional order Model (1) with Neural Networks. Section 4 presents the model training process, along with parameter estimation using real-world COVID-19 data. Finally, the paper concludes with a comprehensive summary and outlines future work in Section 5.

METHODS

Basic Concepts

Numerous fractional derivative definitions have received extensive attention in research. Within this paper, we define the following useful fractional operators(Podlubny 1999):

Definition 1 *The fractional integral* ${}_a\mathcal{D}_t^{-\alpha}$ *of function* $f(t)$ *is defined as follows:*

$${}_a\mathcal{D}_t^{-\alpha}f(t) = \frac{1}{\Gamma(\alpha)}\int_a^t (t-\tau)^{\alpha-1}f(\tau)d\tau, \tag{2}$$

where $\alpha > 0$ and $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the gamma function.

Definition 2 *The Caputo derivative with order* α *of function* $f(t)$ *is given as*

$${}_a^C\mathcal{D}_t^{\alpha}f(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t (t-\tau)^{n-\alpha-1}f^{(n)}(\tau)d\tau, \tag{3}$$

where $n-1 < \alpha < n, n \in Z^+$.

Definition 3 *The Riemann-Liouville derivative with order* α *of function* $f(t)$ *is defined as*

$$ {}_{a}^{RL}\mathcal{D}_t^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dt^n} \int_a^t (t-\tau)^{n-\alpha-1} f(\tau) d\tau, \quad (4) $$

where $n-1 < \alpha < n, n \in Z^+$.

Definition 4 *The Grünwald-Letnikov derivative with order* α *of function* $f(t)$ *is defined as follows:*

$$ {}_{a}^{GL}\mathcal{D}_t^{\alpha} = \lim_{h \to 0 mh=t} h^{-\alpha} \sum_{r=0}^{m} (-1)^r \binom{\alpha}{r} f(t-rh) \quad (5) $$

where $n-1 < \alpha < n$.

The Riemann-Liouville and Caputo definition are connected by the following expressed

$$ {}_{a}^{c}\mathcal{D}_t^{\alpha} = {}_{a}^{RL}\mathcal{D}_t^{\alpha} f(t) - \sum_{k=0}^{n-1} \frac{(t-a)^{k-\alpha} f^{(k)}(a)}{\Gamma(k-\alpha+1)} \quad (6) $$

The Laplace transform of a Caputo fractional derivative with order $\alpha > 0$ is given by (Ren, Sun, and Dai 2016):

$$ \mathfrak{L}\{ {}^{c}\mathcal{D}_t^{\alpha} f(t)\} = s^{\alpha} f(s) - \sum_{k=0}^{n-1} s^{\alpha-k-1} f^{(k)}(0), \quad (7) $$

$n-1 \le \alpha \le n, n \in \mathbb{N}$. It is worthy to mention that

$$ \mathfrak{L}\{ {}^{c}\mathcal{D}_t^{\alpha} f(t)\} = s^{\alpha} f(s) - s^{\alpha-1} f(0), 0 < \alpha \le 1, \quad (8) $$

in which

$$ \mathfrak{L}\{f(t)\} = f(s) = \int_0^{\infty} e^{-st} f(t) dt. \quad (9) $$

Fractional Model

Here, the integer order system (1) is transformed into a fractional order system by using the Caputo fractional-order derivative. Therefore, the fractional-order system is presented for $0 < \alpha \le 1$ thus:

$$ \begin{aligned} {}^{c}\mathcal{D}_t^{\alpha_1}[\mathcal{U}(t)] &= -\rho \frac{\mathcal{U}(t)\mathcal{V}(t)}{N}, \\ {}^{c}\mathcal{D}_t^{\alpha_2}[\mathcal{V}(t)] &= \rho \frac{\mathcal{V}(t)\mathcal{U}(t)}{N} - (\delta+\omega)\mathcal{V}(t), \\ {}^{c}\mathcal{D}_t^{\alpha_3}[\mathcal{P}(t)] &= \delta \mathcal{V}(t), \\ {}^{c}\mathcal{D}_t^{\alpha_4}[\mathcal{Q}(t)] &= \omega \mathcal{V}(t), \end{aligned} \quad (10) $$

subject to

$$ \mathcal{U}(0) = \mathcal{U}_0, \mathcal{V}(0) = \mathcal{V}_0, \mathcal{P}(0) = \mathcal{P}_0, \mathcal{Q}(0) = \mathcal{Q}_0. \quad (11) $$

The COVID-19-free equilibrium (CFE) point of (10) will be obtained and analyzed using the next theorem.

Theorem 1 *The CFE point of systems(10) is* $E_0 = (\mathcal{U}^0, \mathcal{V}^0, \mathcal{P}^0, Q^0) = \left(\frac{\delta+\omega}{\rho}, 0,0,0\right)$.

Proof. First, we set the fractional derivatives to zero, that is

$$ {}^{c}\mathcal{D}_t^{\alpha_1}(\mathcal{U}(t)) = {}^{c}\mathcal{D}_t^{\alpha_2}(F(t)) = {}^{c}\mathcal{D}_t^{\alpha_3}(\mathcal{P}(t)) = {}^{c}\mathcal{D}_t^{\alpha_4}(\mathcal{Q}(t)) = 0. \quad (12) $$

From equation (10) and using $0 = \rho \frac{\mathcal{V}(t)\mathcal{U}(t)}{N} - (\delta+\omega)\mathcal{V}(t)$, we take $\mathcal{V}(t) = 0$, then we solve $\mathcal{U}(t) = \frac{\delta+\omega}{\rho}$ and $\mathcal{V} = 0$. By simplification, we get $\mathcal{P}(t) = 0, \mathcal{Q}(t) = 0$. Thus the CFE point is $E_0 = (\mathcal{U}^0, V^0, \mathcal{P}^0, \mathcal{Q}^0) = \left(\frac{\delta+\omega}{\rho}, 0,0,0\right)$ as required.

Theorem 2 *Let* R_0 *be the Basic Reproduction Number of (10), then CFE is locally asymptotically stable if* $R_0 < 1$ *and unstable if* $R_0 > 1$.

Proof. It can easily be verified via the Next Generation Matrix (Nisar et al. 2021) that basic reproduction number of (10) is $R_0 = \frac{1}{N}$. Also, the Jacobian Matrix of the system (10) is

$$ J = \begin{bmatrix} \frac{-\rho\mathcal{V}}{N} & \frac{-\rho\mathcal{U}}{N} & 0 & 0 \\ \frac{\rho\mathcal{V}}{N} & \frac{\rho\mathcal{U}}{N} - (\delta+\omega) & 0 & 0 \\ 0 & \delta & 0 & 0 \\ 0 & \omega & 0 & 0 \end{bmatrix}, \quad (13) $$

and

$$ J_{E_0} = \begin{bmatrix} 0 & \frac{-(\delta+\omega)}{N} & 0 & 0 \\ 0 & \frac{\delta+\omega}{N} - (\delta+\omega) & 0 & 0 \\ 0 & \delta & 0 & 0 \\ 0 & \omega & 0 & 0 \end{bmatrix}. \quad (14) $$

The characteristics equation of (14) is

$$ det(J-\lambda I) = \begin{vmatrix} -\lambda & \frac{-(\delta+\omega)}{N} & 0 & 0 \\ 0 & \frac{\delta+\omega}{N} - (\delta+\omega) - \lambda & 0 & 0 \\ 0 & \delta & -\lambda & 0 \\ 0 & \omega & 0 & -\lambda \end{vmatrix} = 0. $$

Solving for the eigenvalue is λ, gives

$\lambda = \frac{\delta+\omega}{N} - (\delta+\omega)$ or $\lambda = R_0 - 1$, which shows that $\lambda < 1$ if $R_0 < 1$, as required.

Disease Informed Neural Networks (DINNs)

In this section, the data will be fit to the system of equations and the DINNs framework is necessary because the loss function has two components. The first part represents the mismatch between the network output and the available data, while on the other hand is the residual of the fractional order model. Training neural networks using fractional order model is computationally expensive. It is worth noting that, the Laplace transform of Caputo fractional derivative is

$$ L\{ {}^{c}D_t^{\alpha} v(t)\} = s^{\alpha} v(s) - s^{\alpha-1} v(0), \quad (15) $$

in which $v(s)$ is the Laplace transform of $v(t)$. The fractional derivative ${}^{c}D_t^{\alpha} v(t)$ is approximated using the Laplace transform method and a linearization technique proposed by (Ren et al. 2016).

For $0 < \alpha < 1$, it follows that:

$$ s^{\alpha} \approx \alpha s^1 + (1-\alpha) s^0 = \alpha s + (1-\alpha). \quad (16) $$

By substituting (16) into (15) and taking the inverse Laplace transform, it follows that

$$ {}^{c}D_t^{\alpha} v(t) \approx \alpha v_t + (1-\alpha)[v(t) - v(0)]. \quad (17) $$

Consequently, using equation (17), the fractional order system (10) becomes

$$\begin{aligned}\dot{\mathcal{U}}(t) &= -\frac{\rho}{\alpha_1}\frac{\mathcal{U}(t)\mathcal{V}(t)}{N} - \frac{\alpha_1-1}{\alpha_1}(\mathcal{U}(t)-\mathcal{U}_0),\\ \dot{\mathcal{V}}(t) &= \frac{\rho}{\alpha_2}\frac{\mathcal{V}(t)\mathcal{U}(t)}{N} - (\delta+\omega)\mathcal{V}(t) - \frac{\alpha_2-1}{\alpha_2}(\mathcal{V}(t)-\mathcal{V}_0),\\ \dot{\mathcal{P}}(t) &= \frac{\delta}{\alpha_3}\mathcal{V}(t) - \frac{\alpha_3-1}{\alpha_3}(\mathcal{P}(t)-\mathcal{P}_0),\\ \dot{\mathcal{Q}}(t) &= \frac{\omega}{\alpha_4}\mathcal{V}(t) - \frac{\alpha_4-1}{\alpha_4}(\mathcal{Q}(t)-\mathcal{Q}_0).\end{aligned} \quad (18)$$

The initial conditions remain the same but we recover the classical integer model from equation (18) if $\alpha = 1$. Here, $\boldsymbol{\Psi}_{NN}(\boldsymbol{t};\boldsymbol{\theta}_\Psi) \approx \boldsymbol{\Psi}(\boldsymbol{t})$ represents the approximation of the system of equations (18). The set of parameters denoted by θ are tuned to attain the optimal fit with the data set. If $\boldsymbol{\Psi}_j$ is points, the mean squared error (MSE) is expressed by:

$$MSE_1 = \frac{1}{N}\sum_{j=1}^{N}\left|\hat{\boldsymbol{\Psi}}_{NN}(t_j) - \boldsymbol{\Psi}(t_j)\right|^2, \quad (19)$$

Furthermore, let

$$\begin{aligned}\mathcal{F}_1 &= \dot{\mathcal{U}}(t) - \left(-\frac{\rho}{\alpha_1}\frac{\mathcal{U}(t)\mathcal{V}(t)}{N} - \frac{\alpha_1-1}{\alpha_1}(\mathcal{U}(t)-\mathcal{U}_0)\right),\\ \mathcal{F}_2 &= \dot{\mathcal{V}}(t) - \left(\frac{\rho}{\alpha_2}\frac{\mathcal{V}(t)\mathcal{U}(t)}{N} - (\delta+\omega)\mathcal{V}(t) - \frac{\alpha_2-1}{\alpha_2}(\mathcal{V}(t)-\mathcal{V}_0)\right),\\ \mathcal{F}_3 &= \dot{\mathcal{P}}(t) - \left(\frac{\delta}{\alpha_3}\mathcal{V}(t) - \frac{\alpha_3-1}{\alpha_3}(\mathcal{P}(t)-\mathcal{P}_0)\right),\\ \mathcal{F}_4 &= \dot{\mathcal{Q}}(t) - \left(\frac{\omega}{\alpha_4}\mathcal{V}(t) - \frac{\alpha_4-1}{\alpha_4}(\mathcal{Q}(t)-\mathcal{Q}_0)\right).\end{aligned}$$

then, $\Phi_{NN}(t) = \frac{d\boldsymbol{\Psi}(t)}{dt} - \mathcal{F}(\boldsymbol{\Psi}_{NN}, t; \xi)$. Specifically, $\boldsymbol{\Phi}(t;\boldsymbol{\theta}_\Psi)$ measures how accurate $\Psi_{NN}(t;\theta_\Psi)$ approximates the system (18). Moreover, the assessment of the residual necessitates the computation of the temporal derivative of the neural network's output, a task that can be executed through the application of automatic differentiation (Pollicott, Wang, and Weiss 2012). To measure the difference between the anticipated and actual outcomes, MSE is calculated by:

$$MSE_2 = \frac{1}{N}\sum_{j=1}^{N}\left|\Phi_{NN}(t_j)\right|^2, \quad (21)$$

where N is number of data points. Therefore, the loss function of the DINN scheme is given as:

$$\boldsymbol{L} = MSE_1 + MSE_2. \quad (22)$$

The schematic diagram for the DINN is presented in Figure below:

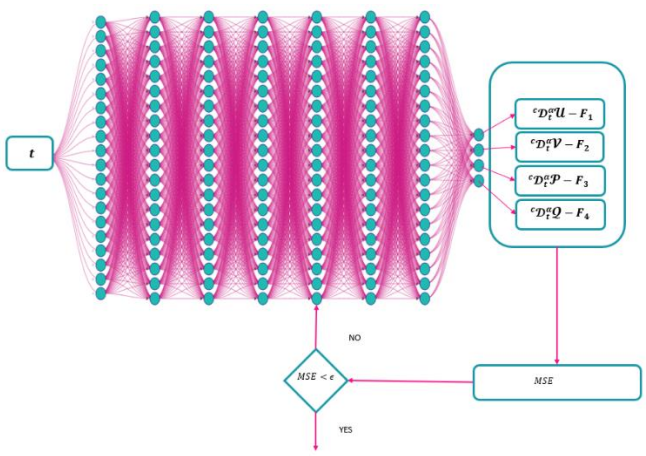

Figure 1: Schematic Diagram of the DINN

RESULTS

Preparation of Data

The numerical experiments were conducted to demonstrate the impact of our model using COVID 19 data from Johns Hopkins Coronavirus Resource Center and WHO reports. The simulations begin on January 22nd, 2020,and end on 15th February 2023 accounting for 1121 days. The summary of the data set is presented in Table (1) below:

Date	Confirmed Global	Deaths Global	Recovered Global
22-01-20	557	17	540
23-01-20	657	18	639
24-01-20	944	26	918
25-01-20	1437	42	1395
⋮	⋮	⋮	⋮
⋮	⋮	⋮	⋮
11-02-23	672824184	6853702	665970482
12-02-23	679900930	6854073	666046857
13-02-23	673037119	6854867	666182252
14-02-23	673216440	6856055	666360385
15-02-23	673443203	6857458	666585745

Table 1: Daily Cumulative COVID-19 Data, total sum from all countries.

Parameters Estimation

The architecture of a neural network model, designed for DINN 10 fully connected layers. It takes a single input feature (representing time) and processes it through a series of fully connected layers with rectified linear activation function (ReLU) activation. The output layer produces predictions for the $\mathcal{U}(t)$, $\mathcal{V}(t)$, $\mathcal{P}(t)$, and $\mathcal{Q}(t)$ compartments of the model.

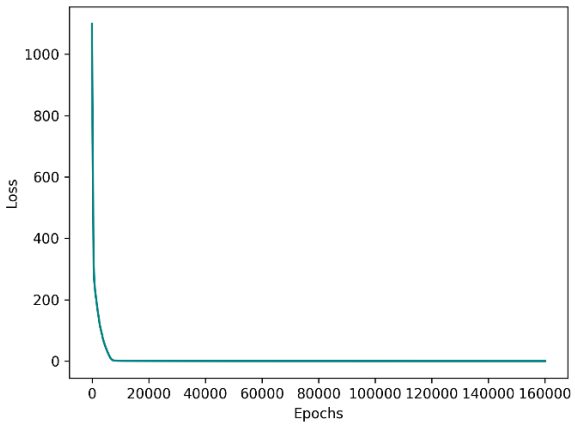

Figure 2: Loss function against number of epochs

The batch size includes the whole time vector. We trained the neural networks on Intel(R) Core(TM) i7-4650U CPU @ 2.30 GHz, RAM 12.0GB 64-bit system. Due to the size of the data and the

complexity of the system, training took 26min 36s with 160,000 epochs to achieve the required accuracy as show in Figure (2). The Adam optimizer (Kingma and Ba 2014) was used, and PyTorch's CyclicLR as learning rate scheduler. The parameter passed are: base_lr=1e-5, max_lr=1e-3, step_size_up=1000, mode="exp_range", gamma=0.85 and cycle_momentum=False.

The fractional orders $\alpha_1, \alpha_2, \alpha_3$, and α_4 have estimated values between 0.5798 and 0.8636 . The search range was considered between 0 and 1.The disease transmission parameter ρ, δ and ω were learned by the neural networks. Specifically, the estimated values of ρ, δ and ω are 0.1730, 0.0466 and 0.0018 , respectively. In order to study the impact of memory effect and nonlocality of the COVID 19 system, we restricted ρ, δ and ω to the learned values in our numerical experimentation. Furthermore, we focused on optimizing the values of $\alpha_1, \alpha_2, \alpha_3$, and α_4 within the specified search range, while keeping ρ, δ, and ω fixed as in Table (2).The experiments revealed that fractional-order parameters have a substantial impact on the system, indicating the existence of memory effects and nonlocal characteristics.

Parameter	Search Range	Minimum Value	Maximum Value	Best Value
α_1	$0 < \alpha_1 \leq 1$	0.5798	1.0000	0.7899
α_2	$0 < \alpha_2 \leq 1$	0.7272	1.0000	0.8636
α_3	$0 < \alpha_3 \leq 1$	0.6992	1.0000	0.8496
α_4	$0 < \alpha_4 \leq 1$	0.7182	1.0000	0.8591
ρ	$0 < \rho \leq 1$	0.1730	0.1730	0.1730
δ	$0 < \delta \leq 1$	0.0466	0.0466	0.0466
ω	$0 < \omega \leq 1$	0.0018	0.0018	0.0018

Table 2: Estimated Parameters

Trained Model

We used the following initial conditions as listed in Table (1): $\mathcal{U}_0 = N - 557, \mathcal{V}_0 = 17, \mathcal{P}_0 = 540$, and $\mathcal{Q}_0 = 0$ where $N = 8025816023$ is the world population as of 15th February 2023. In Figure (3), we have presented the solutions of different classes fitted with the COVID-19 World data for the best values of fractional orders and disease transmission parameters (Figure (2)). Particularly,The subplots represent the susceptible $(\mathcal{U})$, infected $(\mathcal{V})$, recovered $(\mathcal{P})$, and death $(\mathcal{Q})$ cumulative populations.

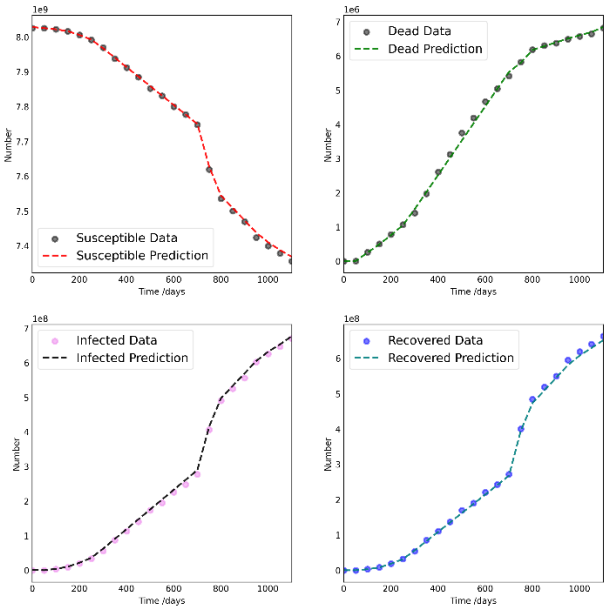

Figure 3: Plot of Raw COVID-19 World Data: Susceptible $(\mathcal{U})$, Infected $(\mathcal{V})$, Recovered $(\mathcal{P})$, and Death $(\mathcal{Q})$ cumulative populations, along with their corresponding fitted solutions

In Figure (4), we explore the vast potentials allowed by increased degrees of freedom of the fractional differential operator, and we examine the dynamic behaviours of the model by plotting the minimum and maximum solutions. The plot allowed us to visualize the disease spectrum of trajectories in response to memory effects and non-localities. By utilizing the entire range of geometrical variations that fractional orders provide, we could gain a deeper understanding of the system's dynamics.

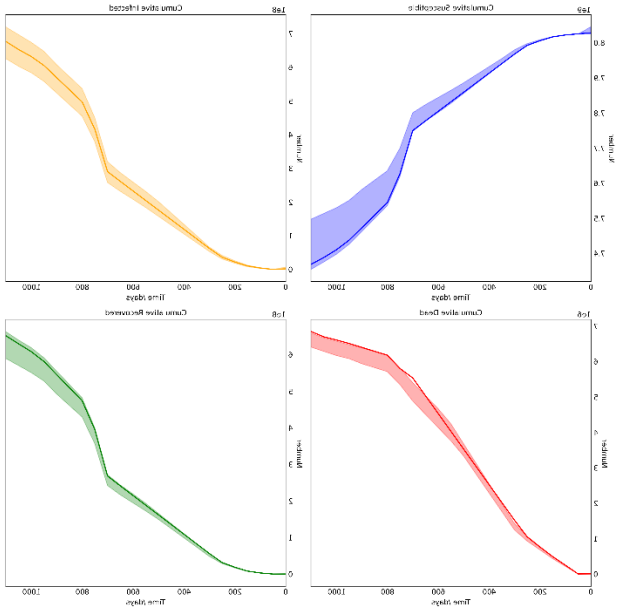

Figure 4: Plot of minimum and maximum solutions to offer a visualization of disease trajectories in response to memory effects and non-localities

CONCLUSION

In this paper, the impact of memory effect and nonlocality in COVID-19 World Data using hybrid fractional order compartmental model and neural networks was investigated. The CFE was obtained and its stability analysis examined. The DINN in relation to hybrid fractional order compartmental model was formulated and used the Laplace Transform to speed up the computation of the fractional derivatives during training of the neural networks. The best values for fractional orders were estimated as $\alpha_1 = 0.7899, \alpha_2 = 0.8636, \alpha_3 = 0.8496$, and $\alpha_4 = 0.8591$ while the disease transmission parameters are $\rho = 0.1730$, $\delta = 0.0466$ and $\omega = 0.0018$. Lastly, the numerical experiments to illustrate the hybrid fractional order compartmental model and neural networks was presented comparing the simulated results with real COVID-19 World Data for all compartments. Also, the dynamic behaviours of the model was examined by plotting the minimum and maximum solutions. In the future, a study of the DINN as explored in this work shall undertaken further but the approach shall involve the utilization of variable fractional order operators.

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Competing Interests

The author declares that there are no conflicts of interest related to this study.

Authors contribution

The author was solely responsible for the conception, design, implementation, data analysis, interpretation, manuscript preparation, and proofreading of this study.

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