

## RESEARCH ARTICLE

**CONJUGATE IMPACTS OF LINEAR VARIABLE THERMAL CONDUCTIVITY AND THERMAL RADIATION ON NONLINEAR ELECTRICALLY CONDUCTING FLUID** Open AccessZayyanu S.Y.<sup>1\*</sup>, Hussaini, A<sup>1</sup> . and Isah, B. Y.<sup>2</sup><sup>1</sup>Department of Mathematics, Sokoto State University, Sokoto, Nigeria.<sup>2</sup>Department of Mathematics, Usmanu Danfodiyo University, Sokoto, Nigeria

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**Abstract**

This study investigates the conjugate impacts of variable thermal conductivity and thermal radiation on nonlinear heat conducting fluid moving vertically through parallel channels.

**Methodology:** The dimensional governing equations were reduced to dimensionless partial differential equations and consequently transformed to ordinary differential equations.

**Results:** The resulting ordinary differential equations were solved using homotopy perturbation method. The study aimed at discovering the possible effects of thermal radiation parameters,  $R$ , thermal conductivity parameter ( $\beta$ ), temperature difference parameter ( $C_T$ ) and Magnetic parameter ( $M$ ), through the help of line graphs.

**Conclusion:** It was found that Velocity and temperature distributions were increasing functions of  $R$ , while Velocity and temperature distributions are decreasing functions of  $\beta$

**KEY WORDS:** *Thermal conductivity, Homotopy perturbation, thermal conductivity, electrically conducting fluid.*

**1.0 INTRODUCTION****Background**

Thermal conductivity is a fundamental property of materials that plays a crucial role in various field of science and engineering. It quantifies the ability of a material to conduct heat. It is useful in designing efficient heat exchangers insulating materials and electronic devices.

Thermal radiation is the electromagnetic radiation emitted by a body due to its temperature. It is a fundamental mode of heat transfer and plays a vital role in various scientific and engineering applications.

Understanding the principles of thermal radiation is crucial for designing energy-efficient systems and optimizing heat exchange processes. Virtually all the fluid of industrial and engineering applications are useful in energy dissipation in a thermal system, for instance, the temperature is known to alter the fluid thermal conductivity significantly, and in control systems it is well known that interference with magnetic field alters the flow behavior of most electrically conducting fluids. This control strategy plays an important role in skin friction and the rate of heat transfer. Other important applications are witnessed in many geophysical situations

where MHD problem arises from the origin of the Earth's magnetic field to the prediction of space weather, the damping of turbulent fluctuations in semiconductor melts during crystal growth and even in the measurement of the flow rates of beverages in the food industry. Other interesting application of MHD are not limited to spraying in metallurgical engineering, electrochemistry and electroplating processing, and other surface occurrences. In view of these wide applications, several researchers have worked on various aspects of Magnetohydrodynamic in the boundary layer region. For instance,

Recent studies have explored the complex dynamics of heat in various fluid systems with variable thermal conductivity and thermal radiation on heat conducting fluids with implications for various engineering and industrial applications. These studies emphasized the importance of variable thermal conductivity in enhancing transport phenomena. These investigations contribute valuable insights to thermal engineering applications, such as heat exchangers, power generation and electronic cooling systems.

The quality of final product depends on thermal management of the system and therefore cooling procedures has to be controlled. Variable thermal conductivity has been shown to increase temperature and thermal boundary layer thickness in Williamson fluid flow over a stretching sheet (Dada and Onwubuoya [1]). Unsteady MHD nanofluid flow with variable thermal conductivity, thermal radiation and viscous dissipation has been investigated for both Blasius and Sakiadis flows (Shayeti and Muzara [2]). The effects of thermal radiation stratification and Joule heating on MHD Sutter by nanofluid flow have been studied, revealing that variable thermal conductivity decreases temperature while radiation decreases it (Sohail *et al.* [3]). Zubair *et al.* [4] Investigated Williamson hybrid nanofluids with variable thermal conductivity and radiation, finding that hybrid nanoparticles enhance energy production compared to single nanostructures. Rehman *et al.* [5] Examined combined convection and thermal radiation in nanofluids with temperature-dependent viscosity and thermal conductivity, observing increased temperature fluctuations with rising thermal conductivity. Ahmed *et al.* [6] Studied MHD flow of Williamson nanofluid over a curved surface with variable thermal conductivity, considering factors such as activation energy and radiation. Islam *et al.* [7] Analyzed radiative aspects of MHD flow in Williamson nanofluid with variable thickness and thermal conductivity, incorporating melting heat flow.

A year after, Bilal *et al.* [8] Investigated Magneto nanofluid flow over a cylinder, considering thermal radiation and variable thermal conductivity, they found that fluid velocity decreases with higher Weissenberg number. Sravan Kumar *et al.* [9] Examined mixed

convective-radiative fluid flow in a channel, considering temperature-dependent thermal conductivity. They analyzed the effects of various parameters on Temperature and Velocity fields. Akbar *et al.* [10] studied the interaction of variable diffusion coefficients with electro kinetically modulated peristaltic motion of a radiative Carreau-Yasuda nanofluid. They observed that Temperature profile increase due to joule heating and manipulation of radiation and thermal conductivity parameter can improve thermal efficiency. Anwar *et al.* [11] Analyzed viscoelastic fluid flow, considering variable thermal conductivity using mathematical models and numerical methods. Rehman *et al.* [12] Conducted comparative analysis of thermo-physical aspects in Magnetized non-Newtonian fluid flow examining the impacts of mixed convection, heat generation and thermal radiation. Abbas *et al.* [13] Explored second-grade fluid flow with Soret and Dufour and thermal radiation. Makinde *et al.* [14] studied a note on hydromagnetic Blasius flow with variable thermal conductivity. Lahmar *et al.* [15] Studied heat transfer of squeezing unsteady nanofluid flow under the effects of magnetic field and variable thermal conductivity.

Jha and Gabriel [16] studied a similarity solution for natural convection flow near a vertical plate with thermal radiation.

The purpose of the present study is to investigate/ examine the conjugate effects of variable thermal conductivity and thermal radiation on non-linear electrically conducting fluid, the study will contribute to the broader field of materials science, enabling researchers to develop new materials with enhanced thermal properties, improving the efficiency of energy conversion and utilization, it also allow engineers and researchers to make more informed decisions in designing a wide range of systems and devices by considering the complex interplay between the variable thermal conductivity and thermal radiation.

## 2.0 MATHEMATICAL ANALYSIS

Consider an infinite vertical channel formed by two parallel plates kept  $h$  distance apart. The channel is filled with an electrically conducting fluid at the expense of radiative heat flux of intensity  $q_r$ , which is absorbed by the plates and transferred to the fluid. Assuming negligible effects of viscous dissipation and assuming all the physical properties of the fluid are constant except for thermal conductivity and thermal radiation. It is also assumed that the flow is at steady-state such a way that  $y$  is dependent on  $y$ .

The  $x'$  — axis is taken along the channel in the vertically upward direction being the direction of the flow, while the  $y'$  — axis is

taken normal to it. The temperature of the plate kept at  $y' = 0$  rise to  $T_w$  and maintained impulsively at uniform velocity  $u = 0$  while the other plate at  $y' = h$  remains at  $T_0$ .

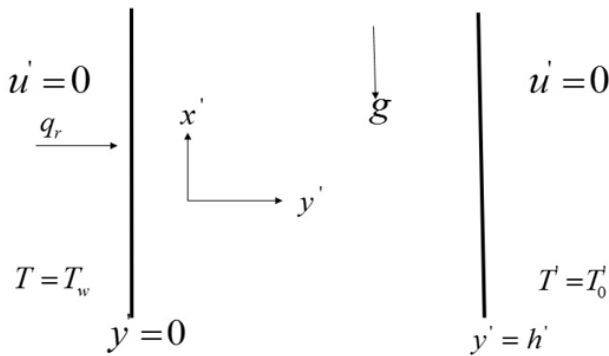


Figure 1: Diagram of the problem

Momentum equation

$$v \frac{\partial^2 u'}{\partial y'^2} + g \beta (T - T_0) - \frac{\sigma \beta_0^2}{\rho} u' = 0 \quad (1)$$

Heat transfer equation

$$\frac{1}{\rho C_p} \left( \frac{\partial}{\partial y'} \left( k \frac{\partial T'}{\partial y'} \right) - \frac{\partial q_r}{\partial y'} \right) = 0 \quad (2)$$

$$\left. \begin{aligned} u' = 0, T = T_w \quad \text{at} \quad y' = T_w \\ u' = 0, T = T_0 \quad \text{at} \quad y' = T_0 \end{aligned} \right\} \quad (3)$$

Using Rosseland approximation

$$q_r = -\frac{4\sigma}{3k'} \frac{\partial T^4}{\partial y'} \quad (4)$$

$$k = k_0(1 + \beta\theta)$$

$$u = \frac{u'}{U}, \quad y = \frac{y'}{h}, \quad U = \frac{1}{g \beta (T_w - T_0)}, \quad \theta = \frac{T' - T_0}{T_w - T_0},$$

$$C_T = \frac{T_w}{T_w - T_0}, \quad R = \frac{4\sigma(T_w - T_0)^3}{3k^* k_0}, \quad k = k_0(1 + \beta\theta) \quad (5)$$

After necessary analysis, the dimensionless equations reduced to Dimensionless Momentum equation

$$\frac{d^2 u}{dy^2} + \theta - M^2 u = 0 \quad (6)$$

Dimensionless Heat transfer equation

$$\left( 1 + \beta\theta + \frac{4}{3} R(C_T + \theta)^3 \right) \frac{d^2 \theta}{dy^2} + 4R(C_T + \theta)^3 \left( \frac{d\theta}{dy} \right)^2 = 0 \quad (7)$$

$$\left. \begin{aligned} u = 0, \theta = 1 \quad \text{at} \quad y = 0 \\ u = 0, \theta = 0 \quad \text{at} \quad y = 1 \end{aligned} \right\} \quad (8)$$

Using homotopy perturbation on equation (7), we have

$$(1-p) \frac{d^2 \theta}{dy^2} + p \left( \frac{d^2 \theta}{dy^2} + \left( \beta\theta + \frac{4}{3} R(C_T + \theta)^3 \right) \frac{d^2 \theta}{dy^2} + 4R(C_T + \theta)^3 \left( \frac{d\theta}{dy} \right)^2 \right) = 0 \quad (9)$$

$$(1-p) \frac{d^2 u}{dy^2} + p \left( \frac{d^2 u}{dy^2} + \theta - M^2 u \right) = 0 \quad (10)$$

But

$$(C_T + \theta)^2 = \theta^2 + 2C_T \theta + C_T^2 \quad (11)$$

$$(C_T + \theta)^3 = \theta^3 + 3C_T \theta^2 + 3C_T^2 \theta + C_T^3 \quad (12)$$

Also

$$\theta = \theta + P\theta_1 + P^2\theta_2 + P^3\theta_3 + \dots \quad (13)$$

$$\theta^2 = \theta_0^2 + 2P\theta_0\theta_1 + 2P^2\theta_0\theta_2 + P^2\theta_1^2 + \dots \quad (14)$$

$$\theta^3 = \theta_0^3 + 3P\theta_0^2\theta_1 + 2P^2\theta_0^2\theta_2 + 2P^2\theta_0\theta_1^2 + \dots$$

$$\left( \frac{d\theta}{dy} \right)^2 = \left( \frac{d\theta_0}{dy} \right)^2 + 2P \frac{d\theta_0}{dy} \frac{d\theta_1}{dy} + 2P^2 \frac{d\theta_0}{dy} \frac{d\theta_2}{dy} + P^2 \left( \frac{d\theta_1}{dy} \right)^2 + \dots \quad (15)$$

Neglecting higher powers of P and substituting Equations (10-14) into Equation (6), we have

$$P^0 : \begin{cases} \frac{d^2 \theta_0}{dy^2} = 0 \\ \frac{d^2 u_0}{dy^2} = 0 \end{cases} \quad (16)$$

$$P^1 : \begin{cases} \frac{d^2 \theta_1}{dy^2} + 4R\theta_0^3 \left( \frac{d\theta_0}{dy} \right)^2 + 12RC_T\theta_0^2 \left( \frac{d\theta_0}{dy} \right)^2 + 12RC_T^2\theta_0 \left( \frac{d\theta_0}{dy} \right)^2 = 0 \\ \frac{d^2 u_1}{dy^2} - M^2 u_0 = -\theta_0 \end{cases} \quad (17)$$

$$P^2 : \begin{cases} \frac{d^2 \theta_2}{dy^2} + \beta\theta_0 \frac{d^2 \theta_1}{dy^2} + \frac{4RC_T^2}{3} \frac{d^2 \theta_1}{dy^2} + \frac{4R\theta_0^2}{3} \frac{d^2 \theta_1}{dy^2} + \frac{8R}{3} \theta_0 \theta_1 \frac{d^2 \theta_0}{dy^2} + \frac{8R}{3} \theta_0 C_T \frac{d^2 \theta_0}{dy^2} + 8R\theta_0^3 \frac{d\theta_0}{dy} \frac{d\theta_1}{dy} + 24RC_T\theta_0^2 \frac{d\theta_0}{dy} \frac{d\theta_1}{dy} + 24R\theta_0\theta_1 C_T \left( \frac{d\theta_0}{dy} \right)^2 + 24RC_T^2\theta_0 \frac{d\theta_0}{dy} \frac{d\theta_1}{dy} + 12RC_T^2\theta_1 \left( \frac{d\theta_0}{dy} \right)^2 = 0 \\ \frac{d^2 u_2}{dy^2} - M^2 u_1 = -\theta_1 \end{cases} \quad (18)$$

The corresponding boundary conditions are

$$\left. \begin{aligned} u_0 = u_1 = u_2 = u_3 = 0 \quad \text{at} \quad y = 0 \\ \theta_0 = 1, \theta_1 = \theta_2 = \theta_3 = 0 \quad \text{at} \quad y = 1 \end{aligned} \right\} \quad (19)$$

Now, Solving the Equations (16-18) and applying the corresponding boundary conditions in (19), we have:

$$\theta_0 = 1 + \alpha y \quad (20)$$

$$\theta_1 = K_5(y^2 - y)\alpha^2 + K_6(y^3 - y)\alpha^3 + K_7(y^4 - y)\alpha^4 + K_8(y^5 - y)\alpha^5 \quad (21)$$

$$\theta_2 = C_{21}y^{12} + C_{20}y^{11} + C_{19}y^{10} + C_{18}y^9 + C_{17}y^8 + C_{16}y^7 + C_{15}y^6 + C_{14}y^5 + C_{13}y^4 + C_{12}y^3 + C_{11}y^2 + C_{10}y + C_9 \quad (22)$$

$$u_0 = 0 \quad (23)$$

$$u_1 = \frac{y^3}{6} - \frac{y^2}{2} + \frac{y}{3} \quad (24)$$

$$u_2 = \frac{K_8 y^7}{42} - \frac{K_7 y^6}{30} + \frac{1}{20} \left( K_6 + \frac{M^2}{6} \right) y^5 - \frac{1}{12} \left( K_5 + \frac{M^2}{2} \right) y^4 - \frac{z_1 y^3}{6} + h_1 y + \frac{M^2 y^3}{18} \quad (25)$$

$$u_3 = L_1 y^{14} + L_2 y^{13} + L_3 y^{12} + L_4 y^{11} + L_5 y^{10} + L_6 y^9 + L_7 y^8 + L_8 y^7 + L_9 y^6 + L_{10} y^5 + L_{11} y^4 + L_{12} y^3 + L_{13} y^2$$

### 3.0 RESULT AND DISCUSSION

The problem on Non-linear heat conducting fluid due to interaction of thermal radiation and linear variable thermal conductivity have been considered. The dimensional partial differential equations are transformed into a system of ordinary differential equation via a Homotopy perturbation technique. The final dimensionless equations are solved analytically and the obtained closed form solution was integrated into Matlab. The default values of pertinent parameters embedded in the flow are Prandtl number ( $Pr = 0.72$ ), thermal radiation ( $R = 0.5$ ), temperature difference parameter ( $C_T = 1.00$ ), Magnetic parameter ( $M = 1.00$ ) and thermal conduction parameter ( $\beta = 0.10$ ).

Fig. 2: Exhibits the impacts of thermal radiation ( $R$ ) on temperature distribution, it is observed that increase in  $R$  gives rise to temperature distribution for both Fig. 2a and Fig. 2b. it is also noted that the impact of  $R$  is more pronounced in Fig. 2b when thermal conduction is absent, this is expected as thermal conduction reduces, the fluid acquire more heat through radiation which in turn increase the temperature of the fluid. Fig. 3 portrays the impact of temperature difference parameter ( $C_T$ ) on temperature distribution, it is evident that the presence of  $C_T$  is inversely proportional to temperature distribution, it is interesting to note that

$C_T$  has no effect on temperature when thermal conduction and thermal radiation are absent.

Fig.4 Depict the effect of Thermal conduction Parameter ( $\beta$ ) on the fluid Temperature, interestingly, thermal boundary layer thickness is retarded, and the fluid Temperature diminishes with a boost in thermal conductivity. The same phenomena is observed in Fig. 5. As thermal conduction parameter ( $\beta$ ) increases, the velocity distribution of the flow is decaying and vice-versa. While Fig. 6. Showcases the role of temperature difference parameter ( $C_T$ ) on velocity distribution, the fluid velocity near the plate is seen to decrease as the temperature difference gets larger, but the velocity far from the plate increase significantly.

Fig. 7 is prepared to show the role of magnetic parameter ( $M$ ) on velocity distribution, it is noted from the Figure that,  $M$  is directly proportional to the fluid velocity and vice-versa. The same trend is observed in Fig. 8. An increase in thermal radiation parameter ( $R$ ) lead to more flow formation which in turn increase the velocity of the fluid.

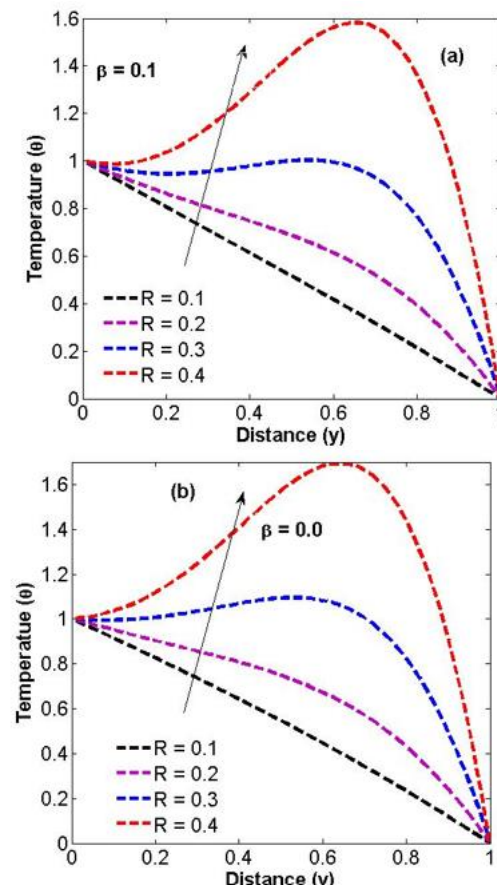


Fig. 2: Effects of Radiation Parameter ( $R$ ) on Temperature Profile

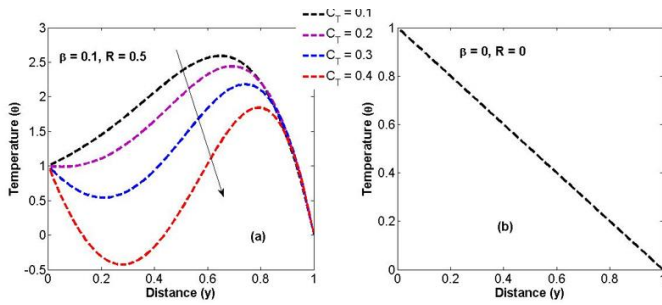


Fig. 3: Effects of Temperature difference parameter ( $C_T$ ) on Temperature Profile

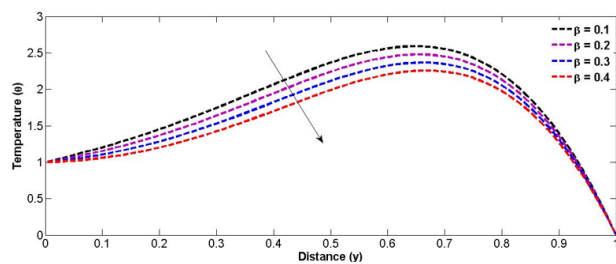


Fig. 4: Effects Variable thermal conductivity Parameter ( $\beta$ ) on Temperature Profile

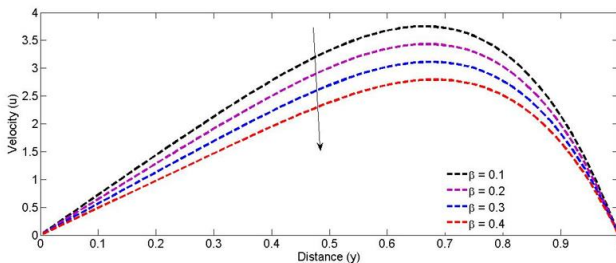


Fig. 5: Effects of Variable thermal conductivity Parameter ( $\beta$ ) on Velocity Profile

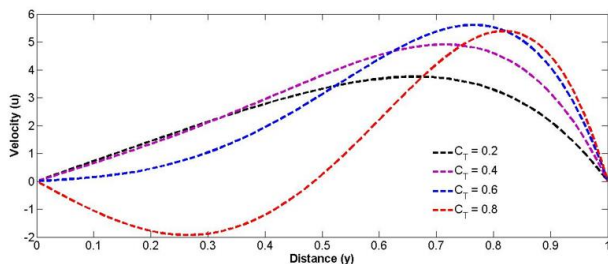


Fig. 6: Effects of Temperature difference parameter ( $C_T$ ) on Velocity Profile

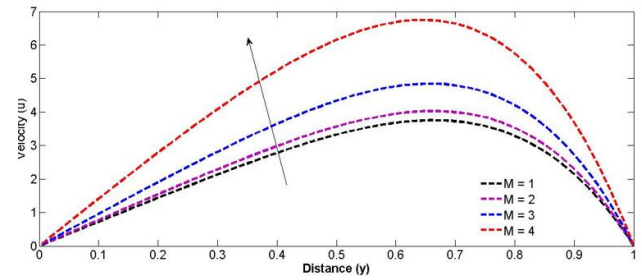


Fig.7: Effects of Magnetic Parameter ( $M$ ) on Velocity Profile

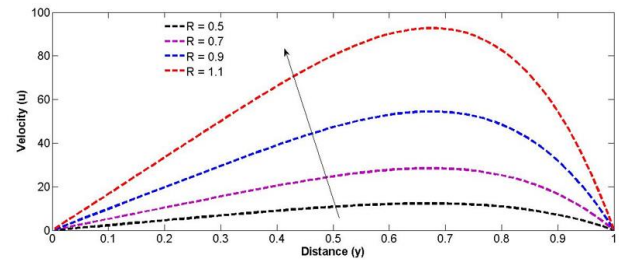


Fig.8: Effects of Radiation Parameter ( $R$ ) on Velocity Profile

#### 4.0 CONCLUSIONS

From the findings of this study, it could be concluded the following situations holds:

- Temperature distribution attain a peak value through an increase in thermal radiation parameter ( $R$ ) when thermal conduction is absence.
- The presence of temperature difference parameter ( $C_T$ ) retard the temperature distribution and velocity distribution at some point.
- Thermal conduction parameter ( $\beta$ ) decreases the temperature and velocity distributions
- Velocity distribution is an increasing function of magnetic ( $M$ ) and thermal radiation parameters ( $R$ )

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